

UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

Pulses in transmission lines

Physics 401, Spring 2019

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illinois.edu

Transmission lines.

Agenda.

- **Distributed parameters network**
- **Pulses in transmission line**
- **Wave equation and wave propagation**
- **Reflections. Resistive load**
- **Thévenin's theorem**
- **Reflection. Non resistive load**
- **Appendix. Error propagation**



Transmission lines.

Main Conceptual Issues:

1. Networks with distributed parameters

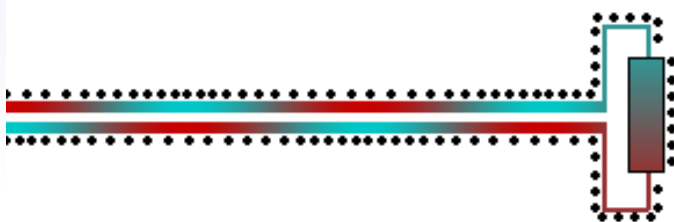
2. Propagation of pulses in transmission lines

3. Impedance matching



Transmission lines. Distributed parameters network.

- Transmission line is a specialized cable designed to carry alternating current of radio frequency, that is, currents with a frequency high enough that its wave nature must be taken into account.



Courtesy Wikipedia

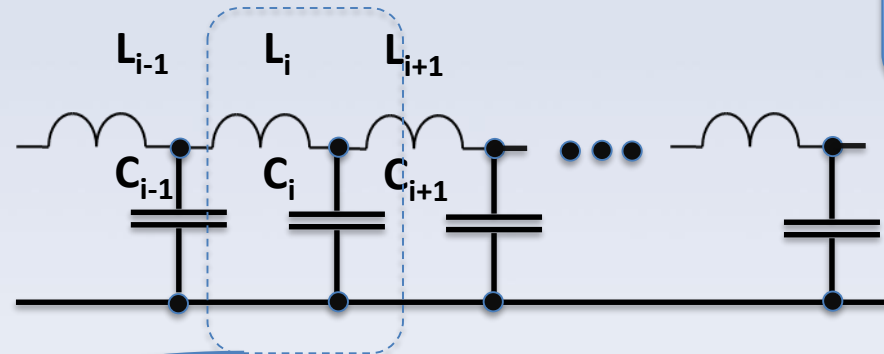


Transmission lines. Distributed parameters network

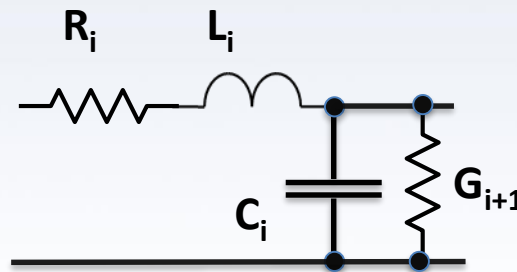
Simplified
equivalent
circuit



Ideal
case



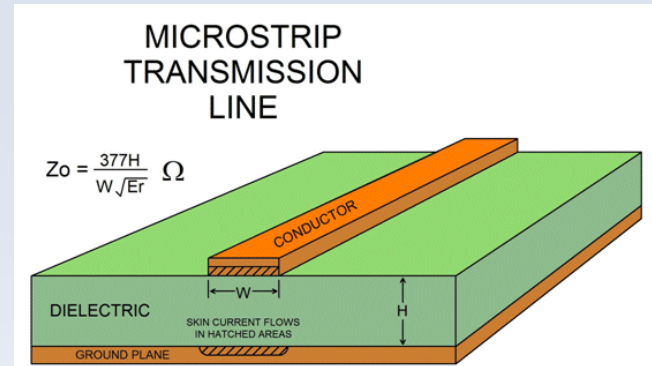
Real
situation



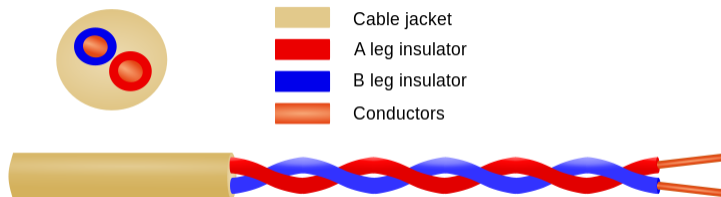
Transmission lines. Different types.



Coaxial cable



Courtesy Analog Devices



Twisted line

Courtesy Wikipedia

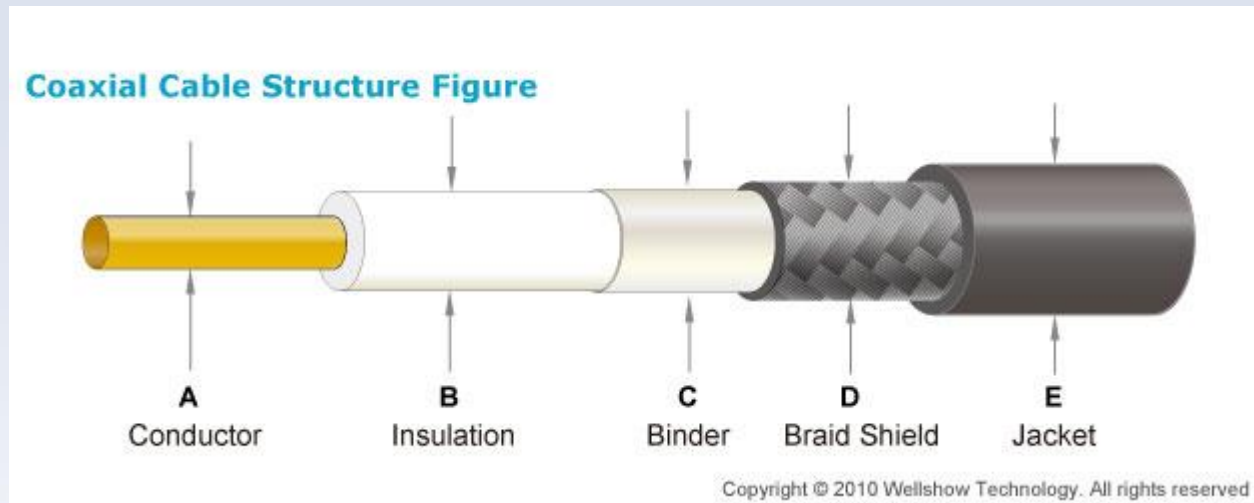


Twin lead

Courtesy Wikipedia



Coaxial cable



Specification:

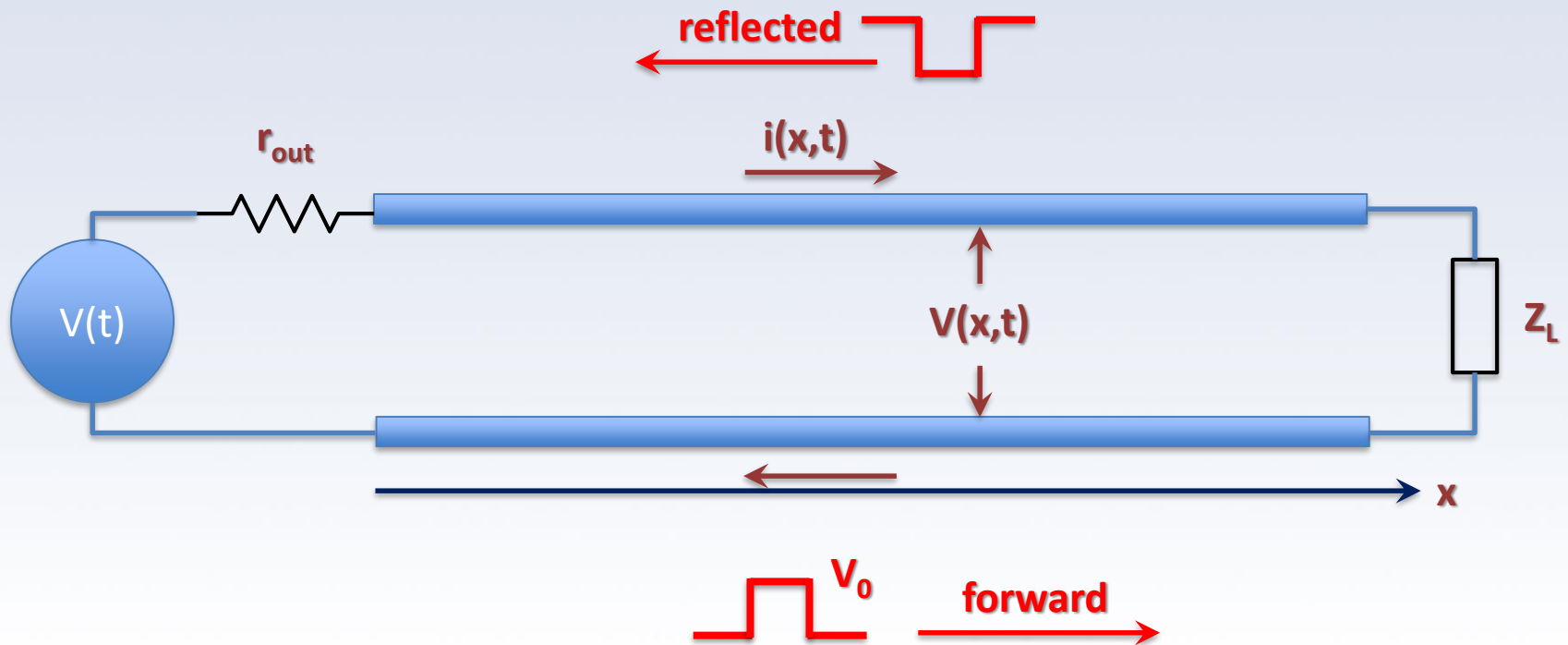
Impedance: 53 Ω

Capacitance: 83 pF/m

Conductor: Bare Copper Wire (1/1.02mm)



Pulses in transmission line



Setup

Wavetek 81

Tektronix 3012B



Sync
output

Triggering
input

Signal
output

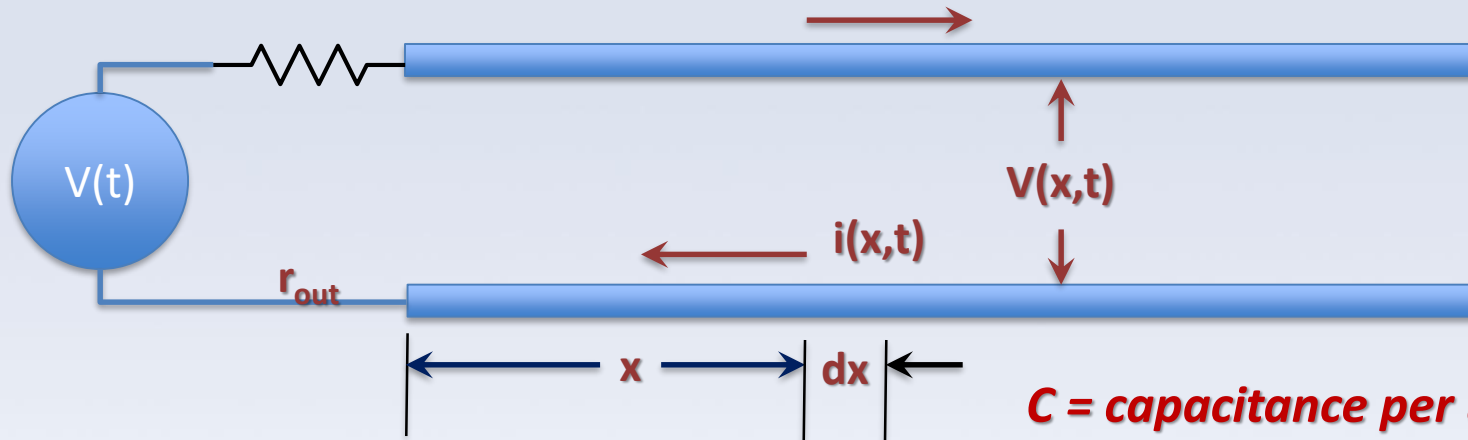
RG8U



Load



The Wave Equation



C = capacitance per unit length
L = inductance per unit length

$$CdxV = -dq;$$

$$C \frac{\partial V}{\partial t} = -\frac{\partial q}{\partial t} = i;$$

$$\frac{\partial i}{\partial x} = -C \frac{\partial V}{\partial t}$$

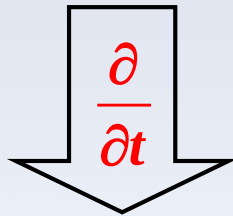
$$dV = -(Ldx) \frac{di}{dt};$$

$$\frac{\partial V}{\partial x} = -L \frac{\partial i}{\partial t}$$



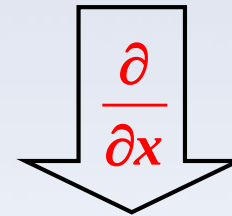
The Wave Equation

$$\frac{\partial i}{\partial x} = -C \frac{\partial V}{\partial t}$$



$$\frac{\partial^2 i}{\partial t \partial x} = -C \frac{\partial^2 V}{\partial t^2} \quad (1)$$

$$\frac{\partial V}{\partial x} = -L \frac{\partial i}{\partial t}$$



$$\frac{\partial^2 V}{\partial x^2} = -L \frac{\partial^2 i}{\partial x \partial t} \quad (2)$$

Combining (1) and (2)

$$\frac{\partial^2 i}{\partial x^2} = LC \frac{\partial^2 i}{\partial t^2}$$

$$\frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t^2}$$



The Wave Equation.

Voltage and current waves.

$$\frac{\partial^2 i}{\partial x^2} = LC \frac{\partial^2 i}{\partial t^2} \quad \frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t^2}$$

Looking for solution

$$V(x, t) = V_0 \sin \omega \left(t - \frac{x}{v} \right)$$

$$i(x, t) = i_0 \sin \omega \left(t - \frac{x}{v} \right)$$

Now substituting $V(x, t)$ and $i(x, t)$ in

$$\frac{\partial V}{\partial x} = -L \frac{\partial i}{\partial t} \quad \frac{\partial i}{\partial x} = -C \frac{\partial V}{\partial t}$$

We can find $V_0 = i_0 \sqrt{\frac{L}{C}}$ or

$$V(x, t) = \sqrt{\frac{L}{C}} i(x, t) = Z_k i(x, t)$$

$$v = \frac{1}{\sqrt{LC}}$$

Speed of wave propagation

Z_k - characteristic Impedance

Equivalent to Ohm's law equation



Characteristic impedance

$$Z_k = \sqrt{\frac{L}{C}}$$

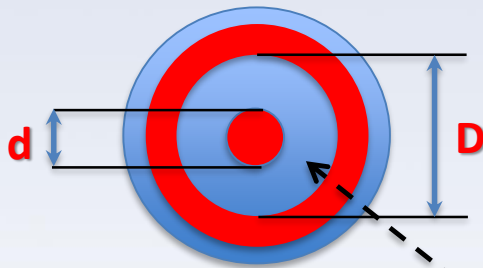
C = capacitance per unit length

L = inductance per unit length

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ (F/m)}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ (H/m)}$$

Cross-section of the coaxial cable



$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln\left(\frac{D}{d}\right)} \text{ (F/m)}$$

$$L = \frac{\mu_0\mu_r}{2\pi} \ln\left(\frac{D}{d}\right) \text{ (H/m)}$$

ϵ_r – dielectric permittivity
 μ_r – magnetic permeability ≈ 1

Finally for coaxial cable:
$$Z_k = \frac{138}{\sqrt{\epsilon_r}} \log\left(\frac{D}{d}\right) \text{ (Ohms)}$$



Speed of wave propagation, delay.

$$v = \frac{1}{\sqrt{LC}}$$

Speed of wave
propagation



$$v = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} \approx \frac{c}{\sqrt{\epsilon_r}}$$

≈ 1

For polyethylene $\epsilon_r \sim 2.25$ (up to 1GHz)

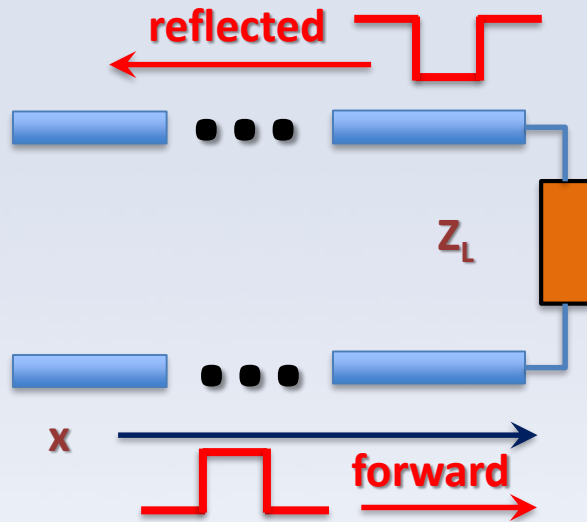
$$\text{Delay time } \tau = \frac{1}{v} (\text{s/m}) \approx 3.336 \cdot 10^{-9} \sqrt{\epsilon_r} (\text{s/m}) = 3.336 \sqrt{\epsilon_r} (\text{ns/m})$$

RG-8/U,
RG58U:

Inner Insulation Materials: Polyethylene
Nominal Impedance: 52 ohm
Delay time $\sim 5\text{ns/m}$



Reflection in transmission line



solution for the traveling in opposite direction

$$\frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t^2}$$

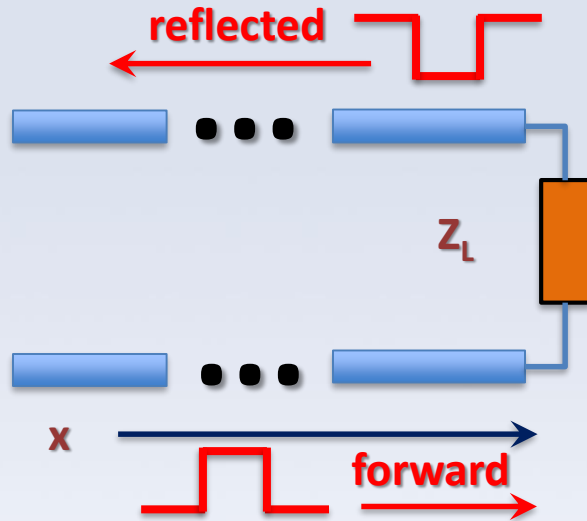
$$V(x, t) = V_0 \sin \omega \left(t + \frac{x}{v} \right)$$

$$i(x, t) = i_0 \sin \omega \left(t + \frac{x}{v} \right)$$

For reflected wave $V_r = -Z_k i_r$



Reflection in transmission line



$$V_r = -Z_k i_r$$

At any point of the transmission line:

$$\frac{V}{i} = R_L$$

$$V = V_r + V_i$$

$$i = i_r + i_i = \frac{V_i}{Z_k} - \frac{V_r}{Z_k}$$

1. Resistive
load $Z_L = R_L$

$$\frac{V_i + V_r}{V_i - V_r} = \frac{R_L}{Z_k} \quad \text{or} \quad V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i$$



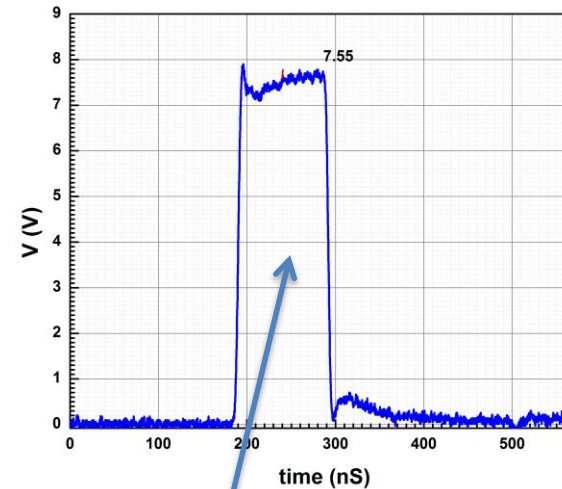
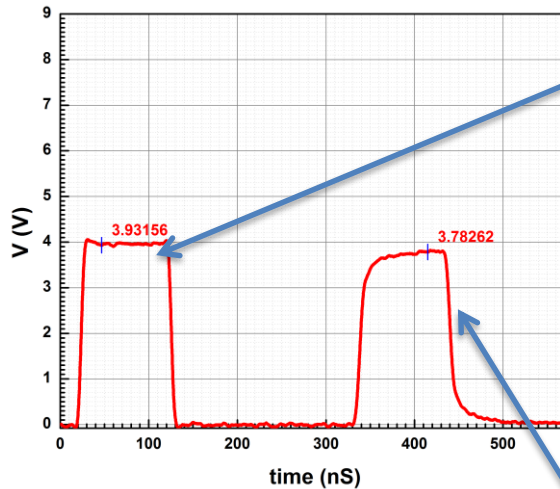
Reflection in transmission line

Resistive load $Z_L = R_L$

$$\frac{V_i + V_r}{V_i - V_r} = \frac{R_L}{Z_k} \quad \text{or} \quad V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i$$

Open line $R_L = \infty \rightarrow V_r = V_i$ and $V_L = V_i + V_r = 2V_i$ (on the load)

Incident pulse



Reflected pulse

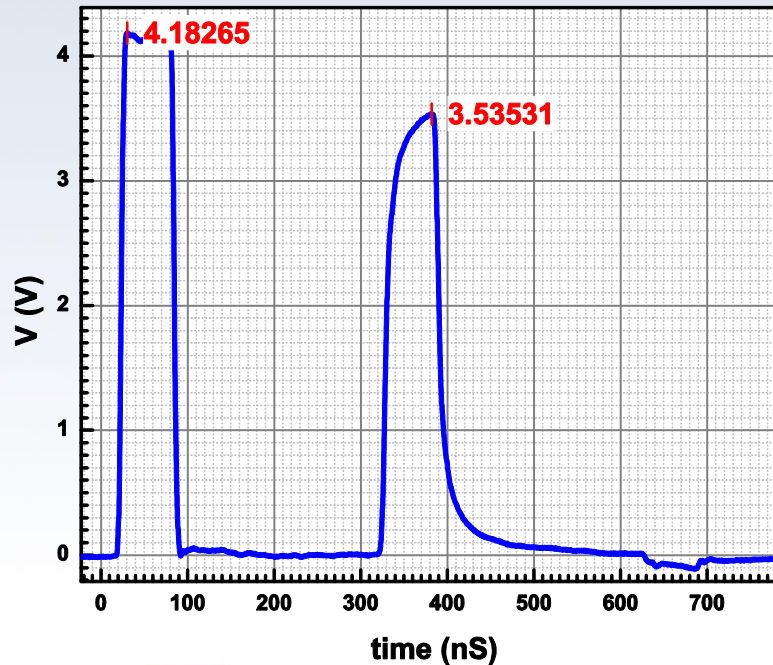
End of the line



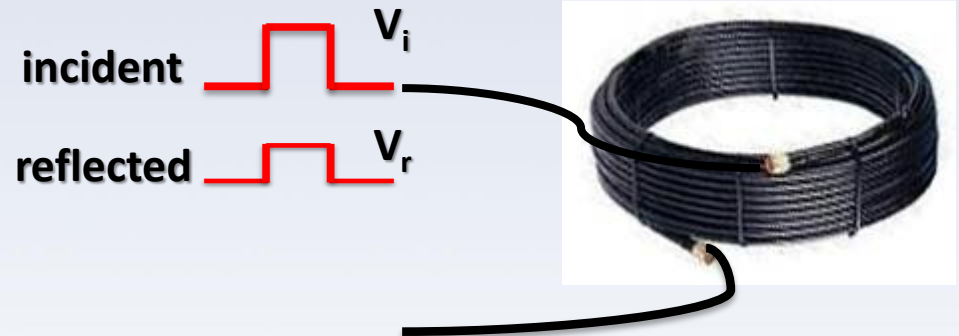
Reflection in transmission line. Loses.

Theory: $R_L = \infty \rightarrow V_r = V_i$

Experiment RG 58U



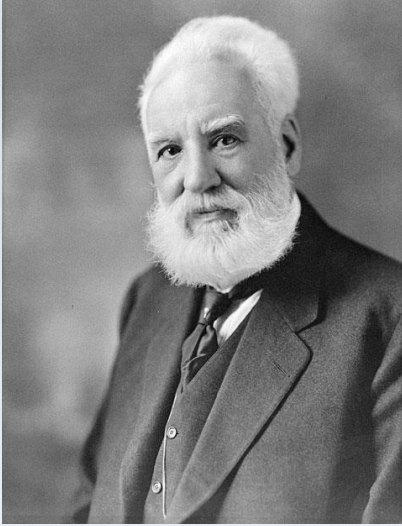
Attenuation (dB per 100 feet)					
MHz	30	50	100	146	150
RG-58U	2.5	4.1	5.3	6.1	6.1



$$ATTN(db) = 20 \log \left(\frac{V_i}{V_r} \right)$$

Important parameter for cable is attenuation per length

Reminder: log units of ration.



Alexander Graham Bell
1847 – 1922)

This unit was named the **bel**, in honor of their founder and telecommunications pioneer **Alexander Graham Bell**

The decibel (dB) is one tenth of the bel (B): 1B = 10dB.

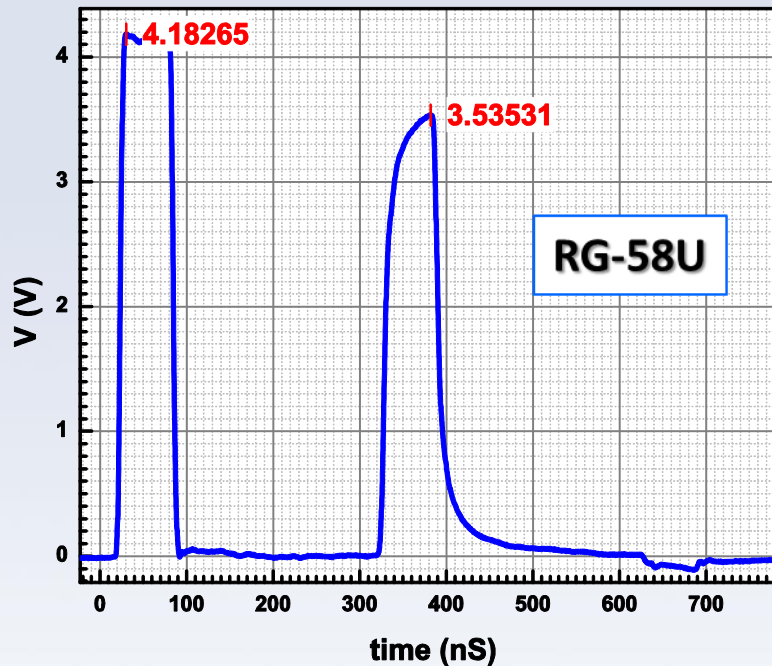
$$L(db) = 10 \log_{10} \left(\frac{P_1}{P_2} \right) \quad \text{power ratio}$$

$$L(db) = 20 \log_{10} \left(\frac{V_1}{V_2} \right) \quad \text{voltage (current, field...) ratio}$$

In case of our transmission line: $ATTN(db) = 20 \log \left(\frac{V_i}{V_r} \right)$

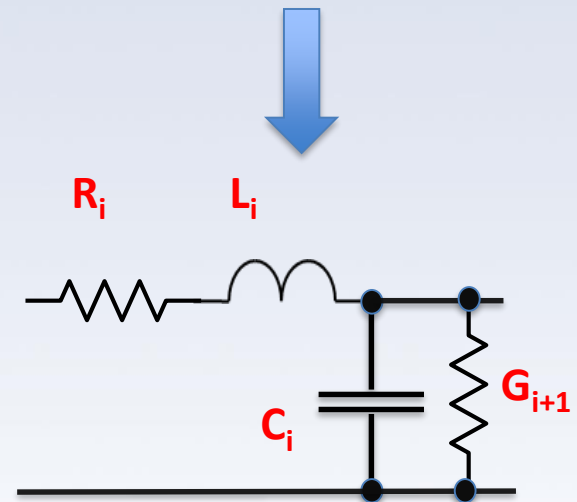


Reflection in transmission line. Loses.

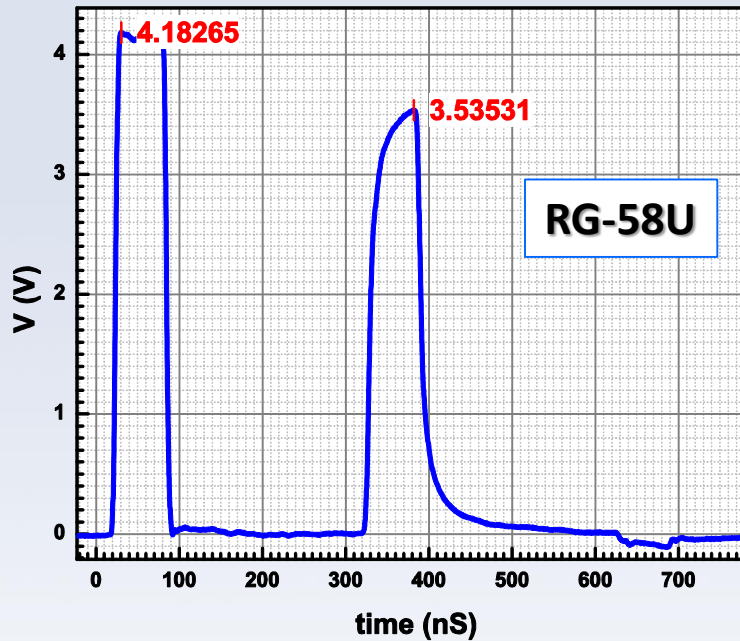


In our case: $Attn(200\text{ ft}) = 20 \log \left(\frac{4.18}{3.54} \right) \approx 1.46\text{dB}$

Where it is coming from?

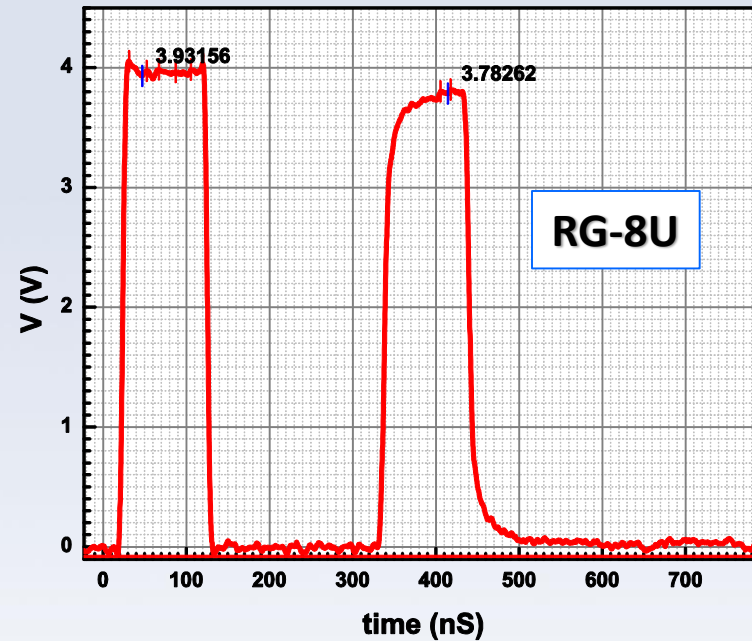


Different cables loses.



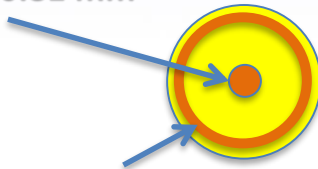
$$Attn(200\text{ ft}) = 20 \log \left(\frac{4.18}{3.54} \right) \approx 1.46\text{dB}$$

>



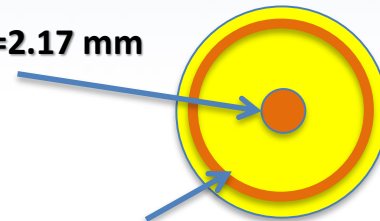
$$Attn(200\text{ ft}) = 20 \log \left(\frac{3.932}{3.78} \right) \approx 0.335\text{dB}$$

Core $\phi=0.81\text{ mm}$



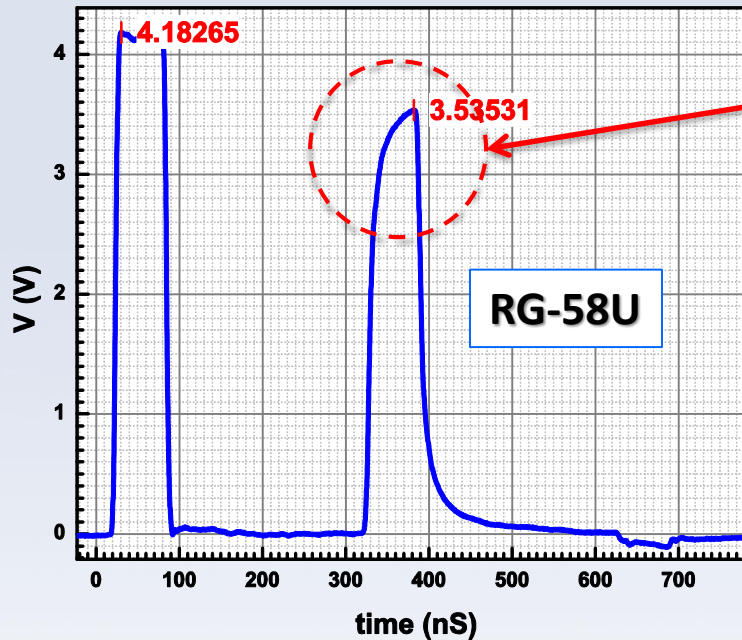
Dielectric $\phi=2.9\text{ mm}$

Core $\phi=2.17\text{ mm}$



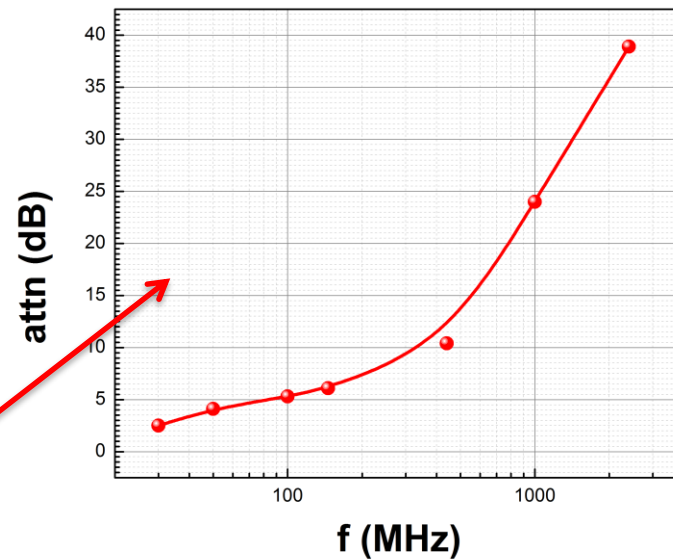
Dielectric $\phi=7.2\text{ mm}$

Losses. Frequency dispersion.

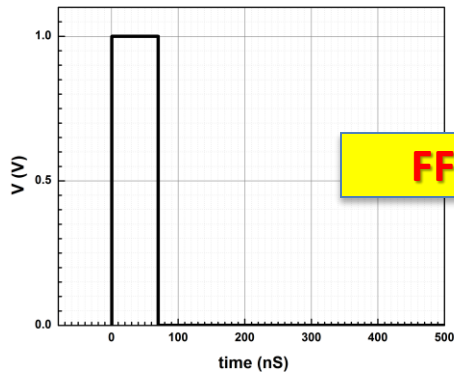


Reflected pulse does not follow the shape of the incident pulse

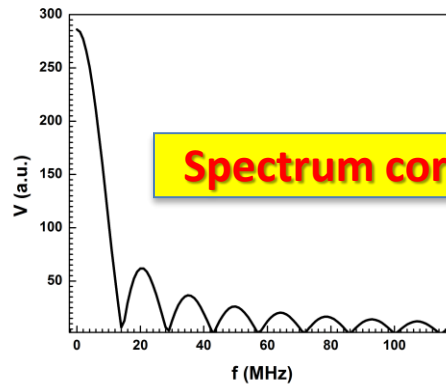
Frequency dependence of the attenuation RG-58U cable



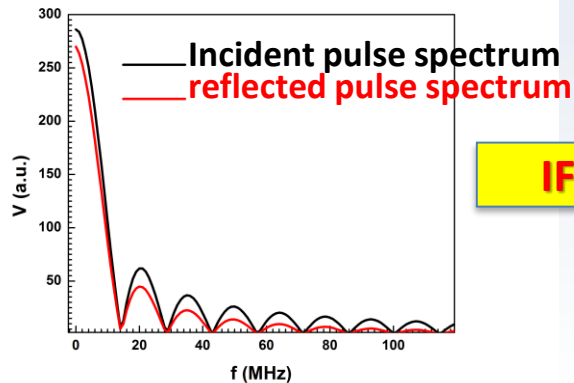
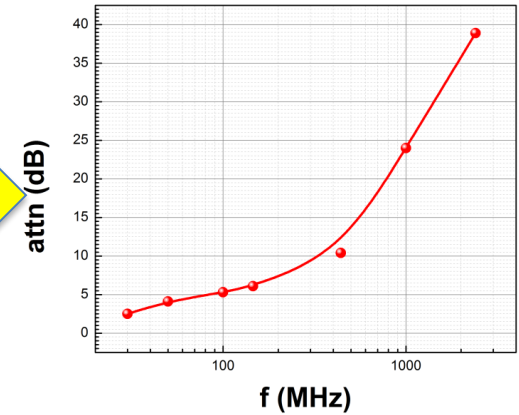
Losses. Frequency dispersion.



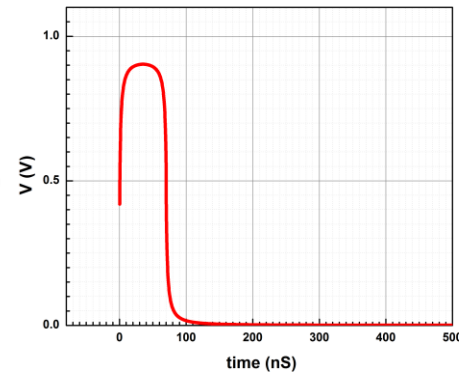
FFT



Spectrum correction



IFFT

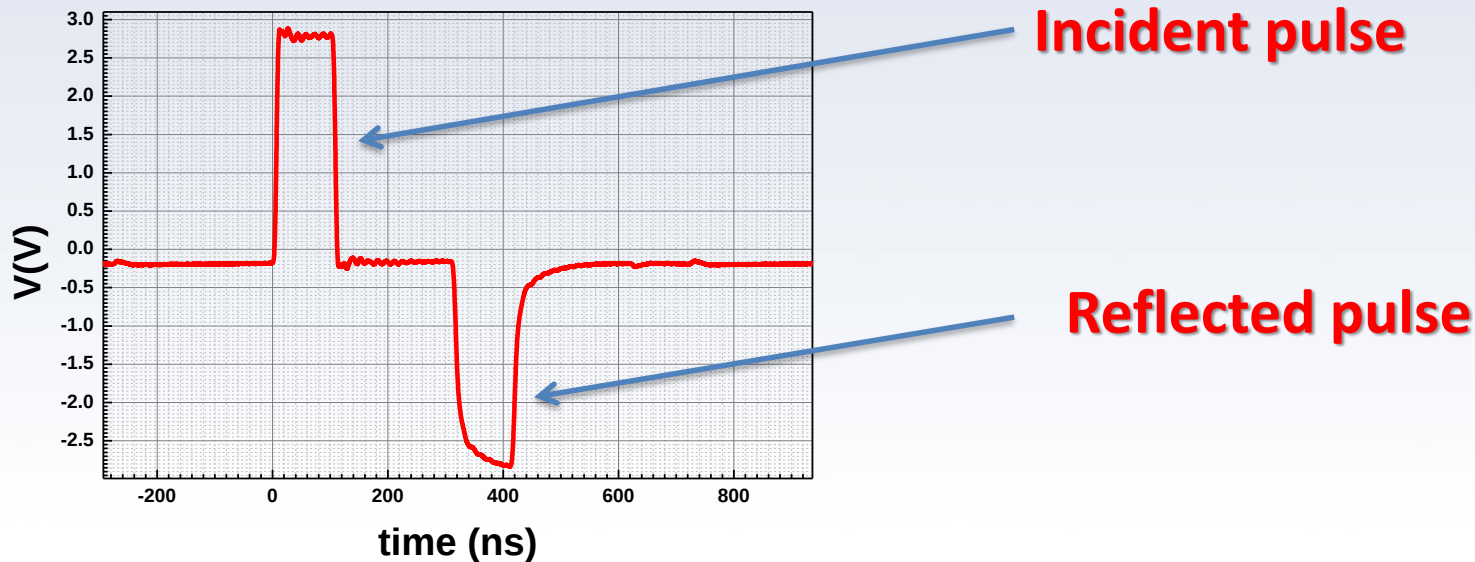


Reflection in transmission line

Resistive load $Z_L = R_L$

$$\frac{V_i + V_r}{V_i - V_r} = \frac{R_L}{Z_k} \quad \text{or} \quad V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i$$

Shorted line $R_L = 0 \rightarrow V_r = -V_i$



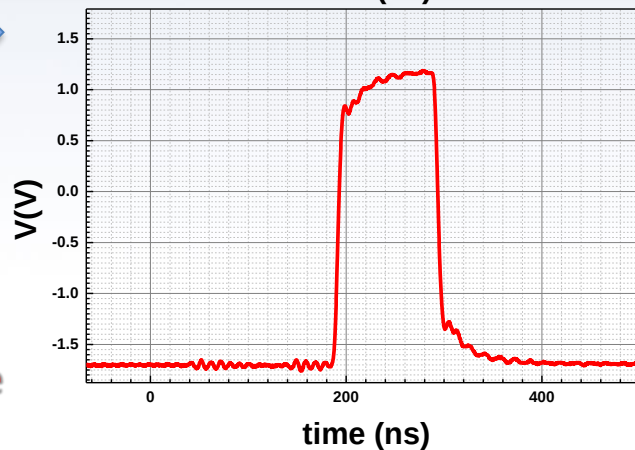
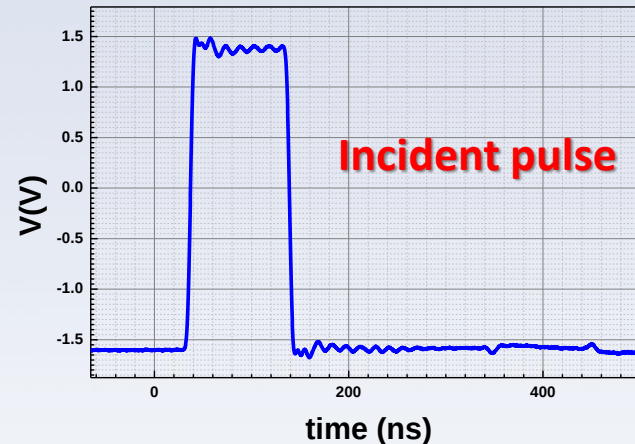
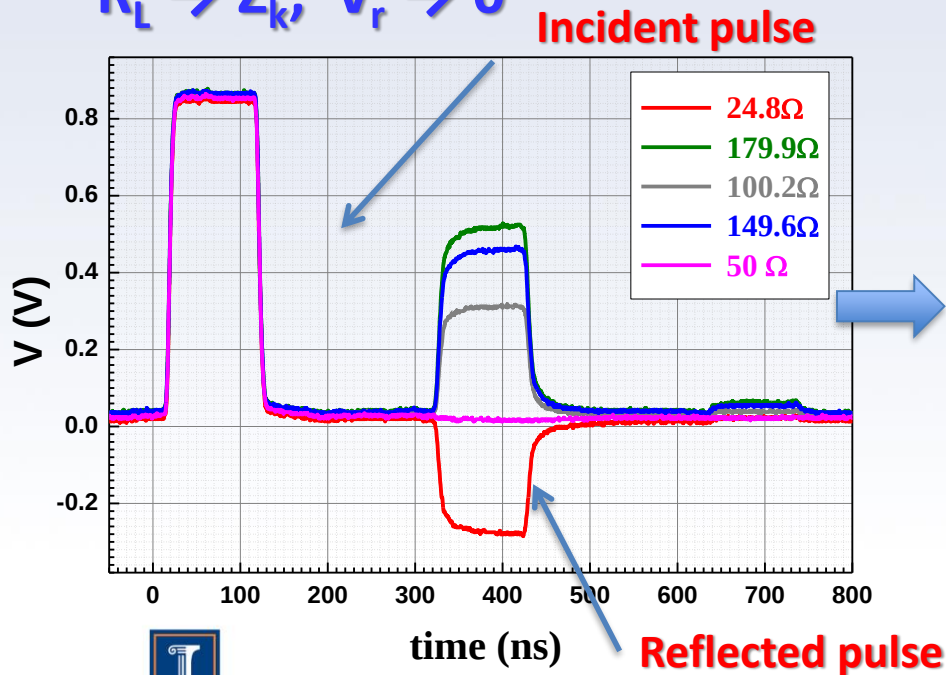
Reflection in transmission line.

Resistive load $Z_L = R_L$

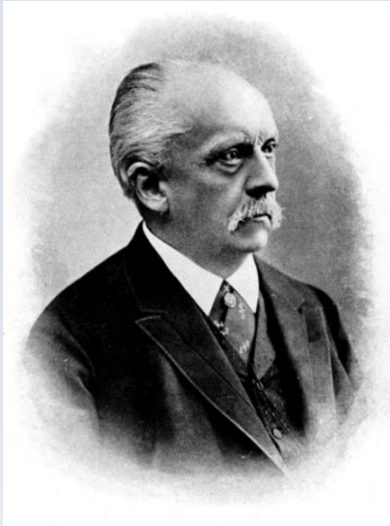
$$\frac{V_i + V_r}{V_i - V_r} = \frac{R_L}{Z_k} \quad \text{or} \quad V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i$$

Matching the load impedance

$R_L \Rightarrow Z_k; V_r \rightarrow 0$



Thévenin's theorem

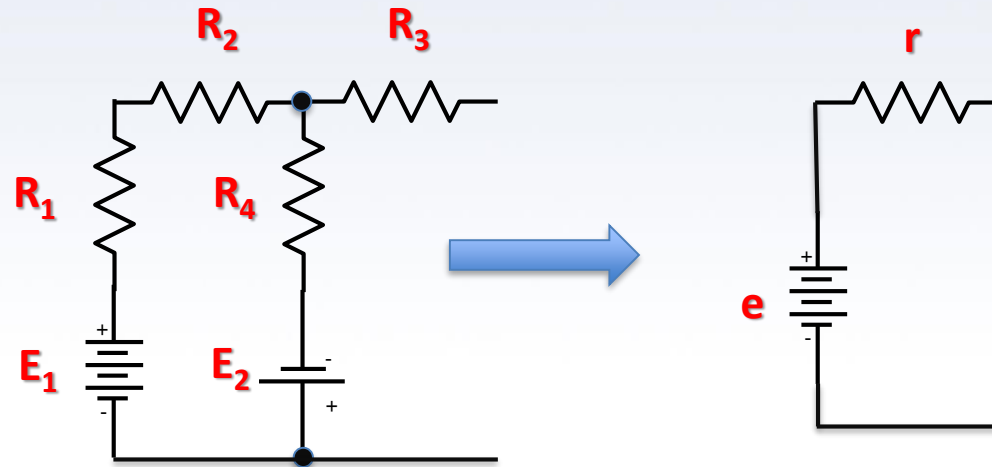


Hermann Ludwig
Ferdinand von Helmholtz
(1821-1894)



Léon Charles Thévenin
(1857–1926)

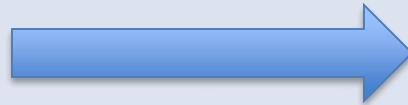
Any combination of batteries and resistances with two terminals can be replaced by a single voltage source e and a single series resistor r



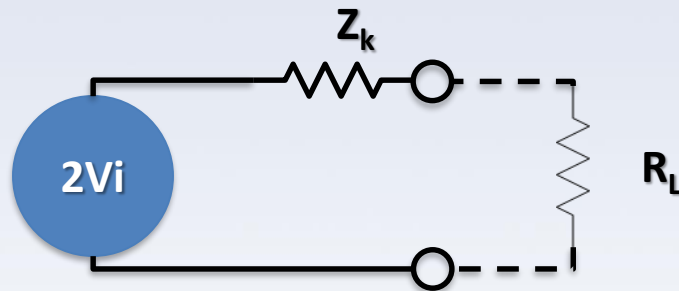
Thévenin's theorem. Transmission line.

$$V = V_r + V_i = i \bullet R_L$$

$$i = i_r + i_i = \frac{V_i}{Z_k} - \frac{V_r}{Z_k}$$



$$i = \frac{2V_i}{R_L + Z_k}$$



From this equivalent equation we can find the maximum possible power delivered to R_L

$$P = i^2 R_L = \frac{(2V_i)^2}{(R_L + Z)^2} R_L$$

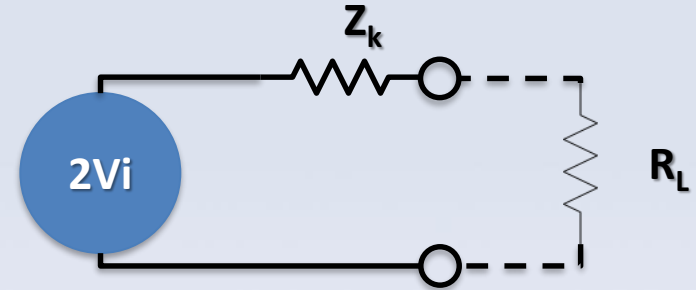
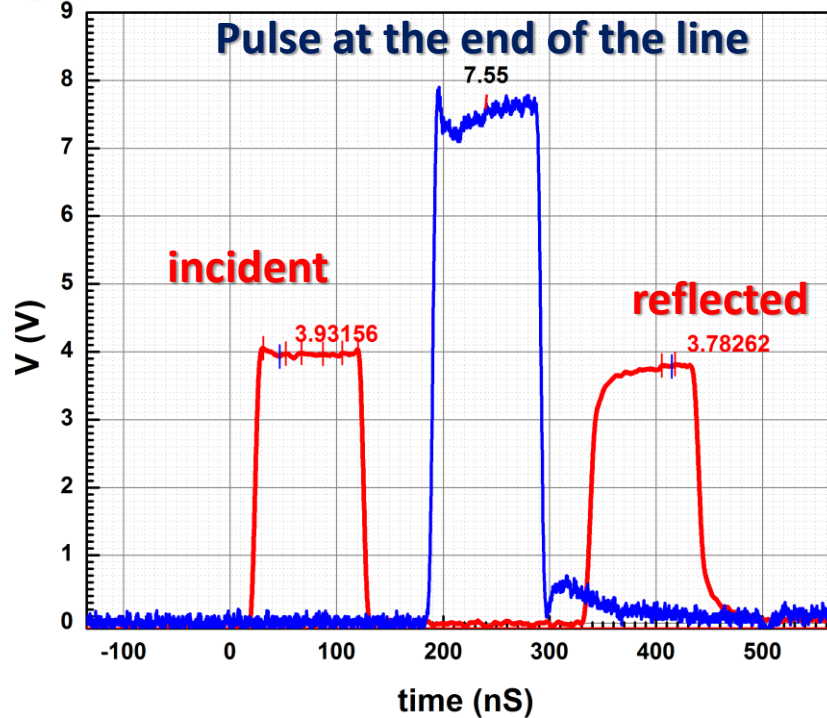


$P = P_{\max}$ if $R_L = Z_k$ (no reflection)



Thévenin's theorem. Experiment.

RG 8U



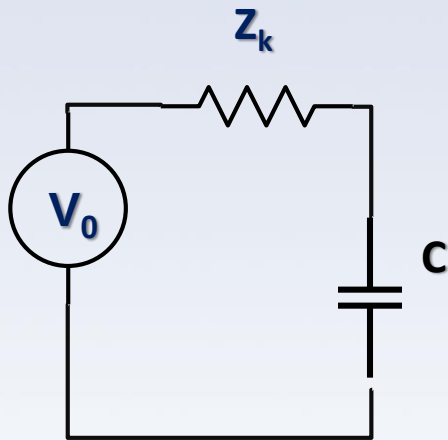
This experiment better to perform on RG 8U cable because of lower attenuation

$R_L = \infty$, amplitude of the pulse at the end of line is expected to be $2V_i$, where V_i is the amplitude of the incident pulse



Reflection. Capacitive load. Experiment

$$i = \frac{2V_i}{Z_L + Z_k}$$

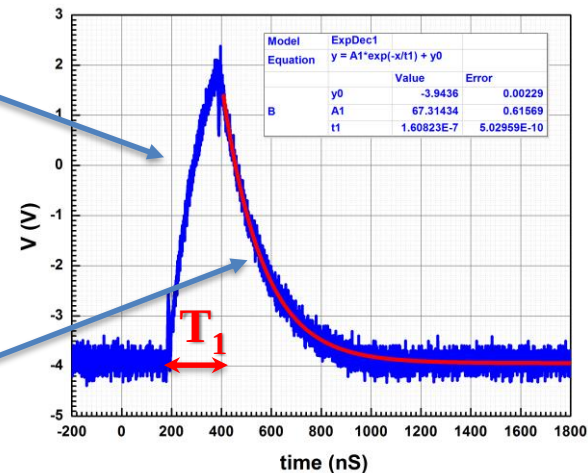
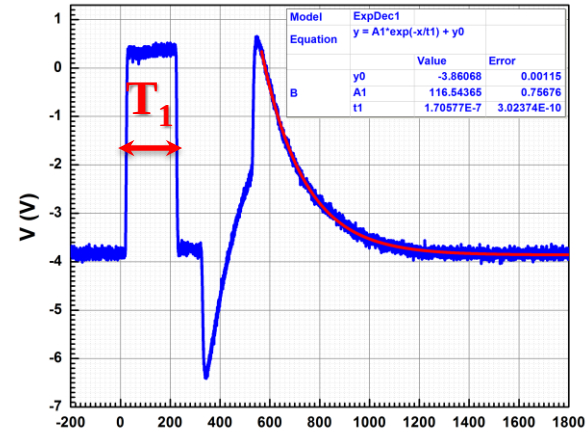


$$\tau = Z_k C$$

$$C = \frac{\tau}{Z_k} \approx 3.2 \text{ nF}$$

$$V_L = \left[1 - \exp\left(\frac{-t}{\tau}\right) \right]$$

$$V_L = 2V_i \left[1 - \exp\left(\frac{-T_1}{\tau}\right) \right] \left[\exp\left(\frac{-(t - T_1)}{\tau}\right) \right]$$

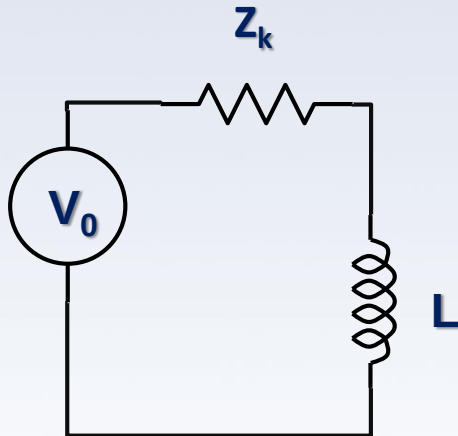


Reflection. Inductive load. Experiment

$$i = \frac{2V_i}{Z_L + Z_k}$$

$$2V_i = iZ_k - L \frac{di}{dt};$$

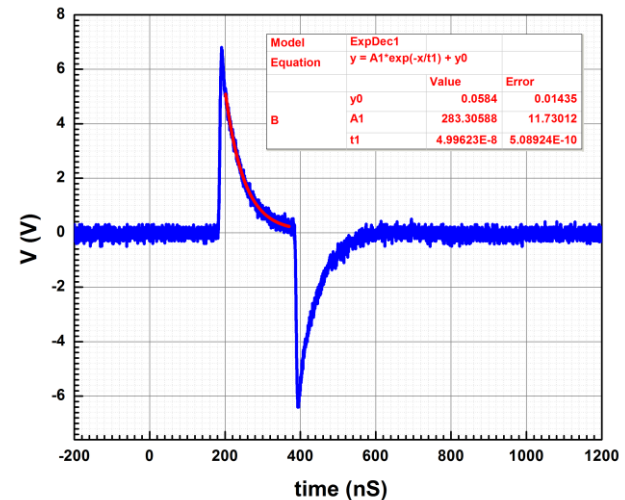
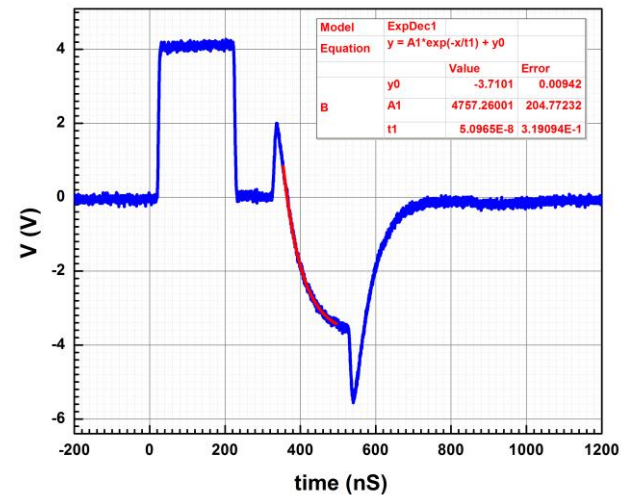
$$i = i_0 \left(1 - \exp\left(\frac{-t}{\tau}\right) \right);$$



$$\tau = \frac{L}{Z_k}$$

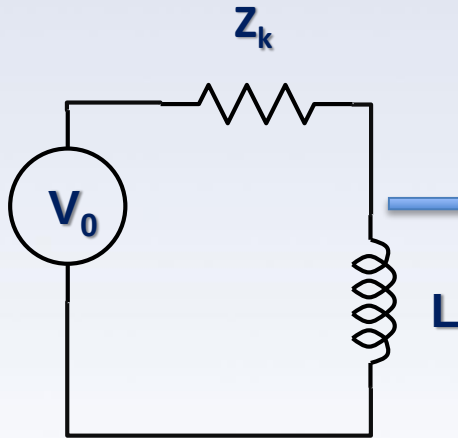
$$\tau \cong 50\text{ns},$$

$$L = \tau Z_k \sim 2.5\mu\text{H}$$



Reflection. Inductive load.

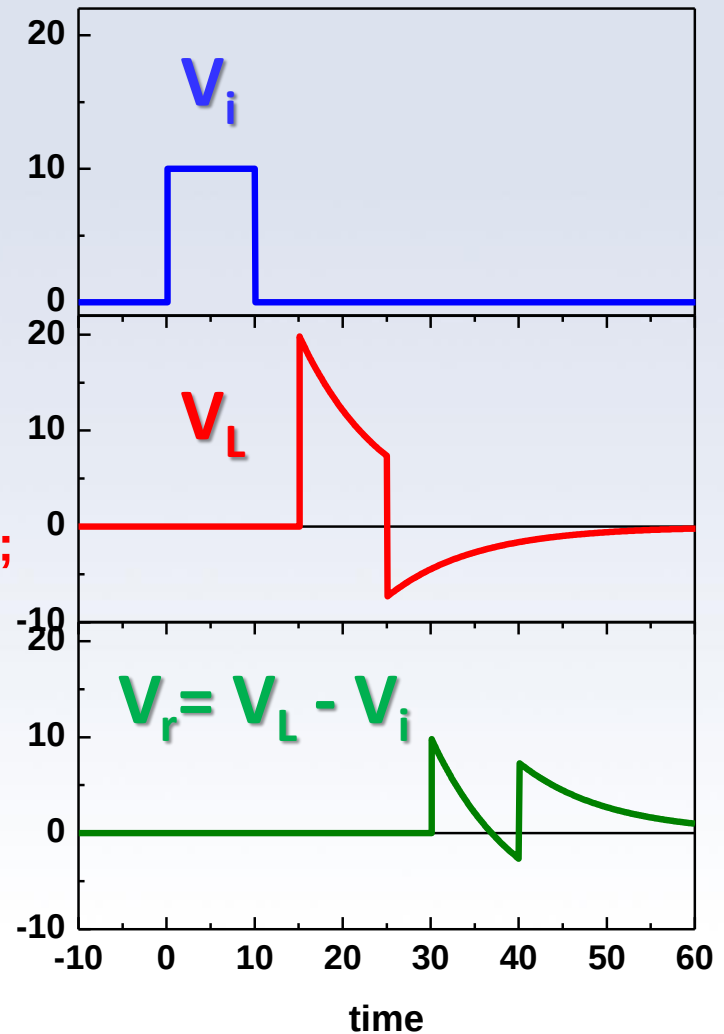
$$i = \frac{2V_i}{Z_L + Z_k}$$



$$2V_i = iZ_k - L \frac{di}{dt};$$

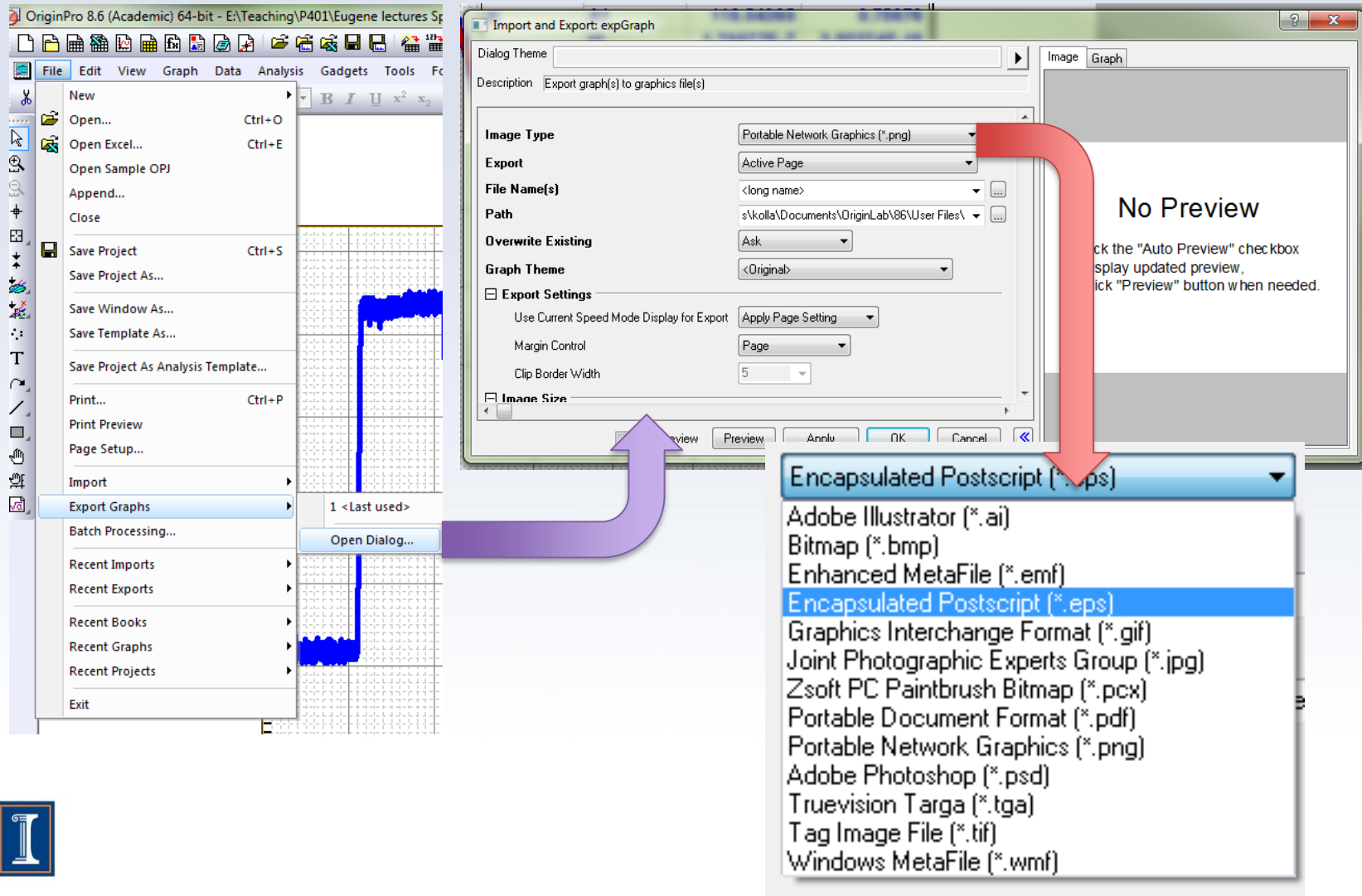
$$i = i_0 \left(1 - \exp\left(\frac{-t}{\tau}\right) \right);$$

$$\tau = \frac{L}{Z_k}$$



Appendix #1.

Export graphs from Origin



Appendix #2.

Reminders

1. The reports should be uploaded to the **proper** folder and **only** to the proper folder

For example folder ***Frequency domain analys_L1*** should be used by students from L1 section only

I would recommend the file name style as:

L1_lab3_student1

Lab section

Lab number

Your name

You do not need to submit two copies in pdf and in MsWord formats

2. Origin template for this week Lab:

\\engr-file-03\phyinst\APL Courses\PHYCS401\Common\Origin templates\Transmission line\Time trace.otp

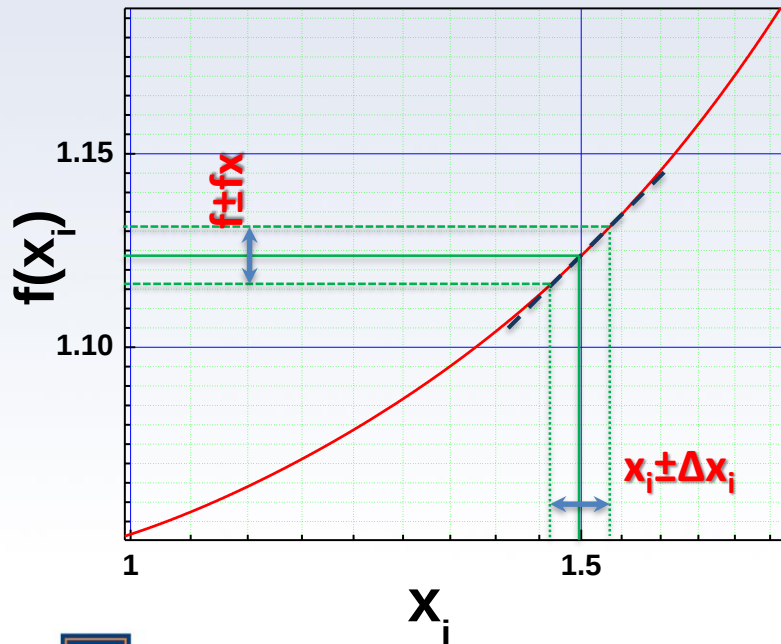


Appendix #3.

Error propagation.

$$y = f(x_1, x_2 \dots x_n)$$

$$\Delta f(x_i, \Delta x_i) = \sqrt{\sum_{i=1}^n \left[\frac{\partial f}{\partial x_i} \right]^2 \cdot \Delta x_i^2}$$



Error propagation. Example.

Derive resonance frequency f
from measured inductance
 $L \pm \Delta L$ and capacitance $C \pm \Delta C$

$$f_0(L, C) = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

$$L_1 = 10 \pm 1\text{mH}, \quad C_1 = 10 \pm 2\mu\text{F}$$

$$\Delta f(L, C, \Delta L, \Delta C) = \sqrt{\left[\frac{\partial f}{\partial L}\right]^2 \cdot \Delta L^2 + \left[\frac{\partial f}{\partial C}\right]^2 \Delta C^2}$$

$$\frac{\partial f}{\partial L} = \frac{-1}{4\pi} C^{-\frac{1}{2}} L^{-\frac{3}{2}};$$

$$\frac{\partial f}{\partial C} = \frac{-1}{4\pi} L^{-\frac{1}{2}} C^{-\frac{3}{2}}$$

Results:

$$f(L_1, C_1) = 503.29212104487\text{Hz}$$

$$\Delta f = 56.26977\text{Hz}$$

$$f(L_1, C_1) = 503 \pm 56\text{Hz}$$



Error propagation. Example.

$L_1 = 10 \pm 1\text{mH}$, $C_1 = 10 \pm 1\mu\text{F}$ Where these numbers are coming from?

1. Using commercial resistors, capacitors, inductances...



$C=500\text{pF} \pm 5\%$



$L=35\text{mH} \pm 10\%$

2. Measuring the parameters using standard equipment

SENCORE “Z” meter model LC53

Capacitance measuring accuracy $\pm 5\%$

Inductance measuring accuracy $\pm 2\%$



Agilent E4980A Precision LCR Meter

Basic accuracy $\pm 0.05\%$



Appendix #4.

Nonlinear fitting. Main idea

Origin uses the **Levenberg–Marquardt** algorithm for nonlinear fitting

From experiment you have the array (x_i, y_i) of independent and dependent variables: x_i (e.g. f- frequency) and y_i (e.g. magnitude of the signal) and you have to optimize the vector of fitting parameters β of your model function $f(x, \beta)$ in order to minimize the sum of squares of deviations:

$$S(\beta) = \sum_{i=1}^m [y_i - f(x_i, \beta)]^2$$

Important point is the choice of fitting parameters. In some cases the algorithm will work with $\beta=(1,1...1)$, but in many situations the choice of more realistic parameters will lead to solution

For details go to:

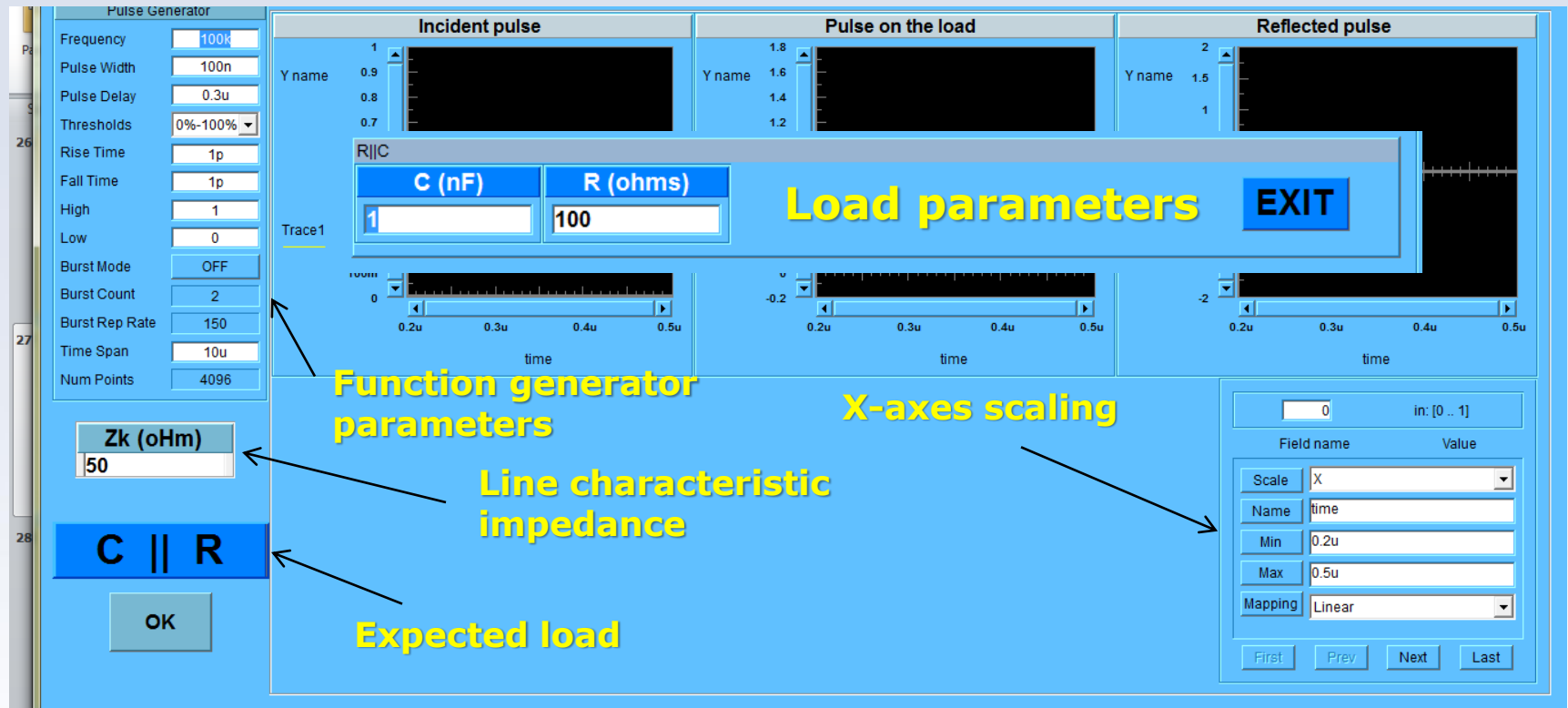
[http://en.wikipedia.org/wiki/Levenberg–Marquardt_algorithm](http://en.wikipedia.org/wiki/Levenberg%E2%80%93Marquardt_algorithm)

K. Levenberg. "A Method for the Solution of Certain Non-Linear Problems in Least Squares". The Quarterly of Applied Mathematics, 2: 164-168 (1944).



Appendix #5. Unknown Load Simulation

- Transmission line. Unknown load simulation

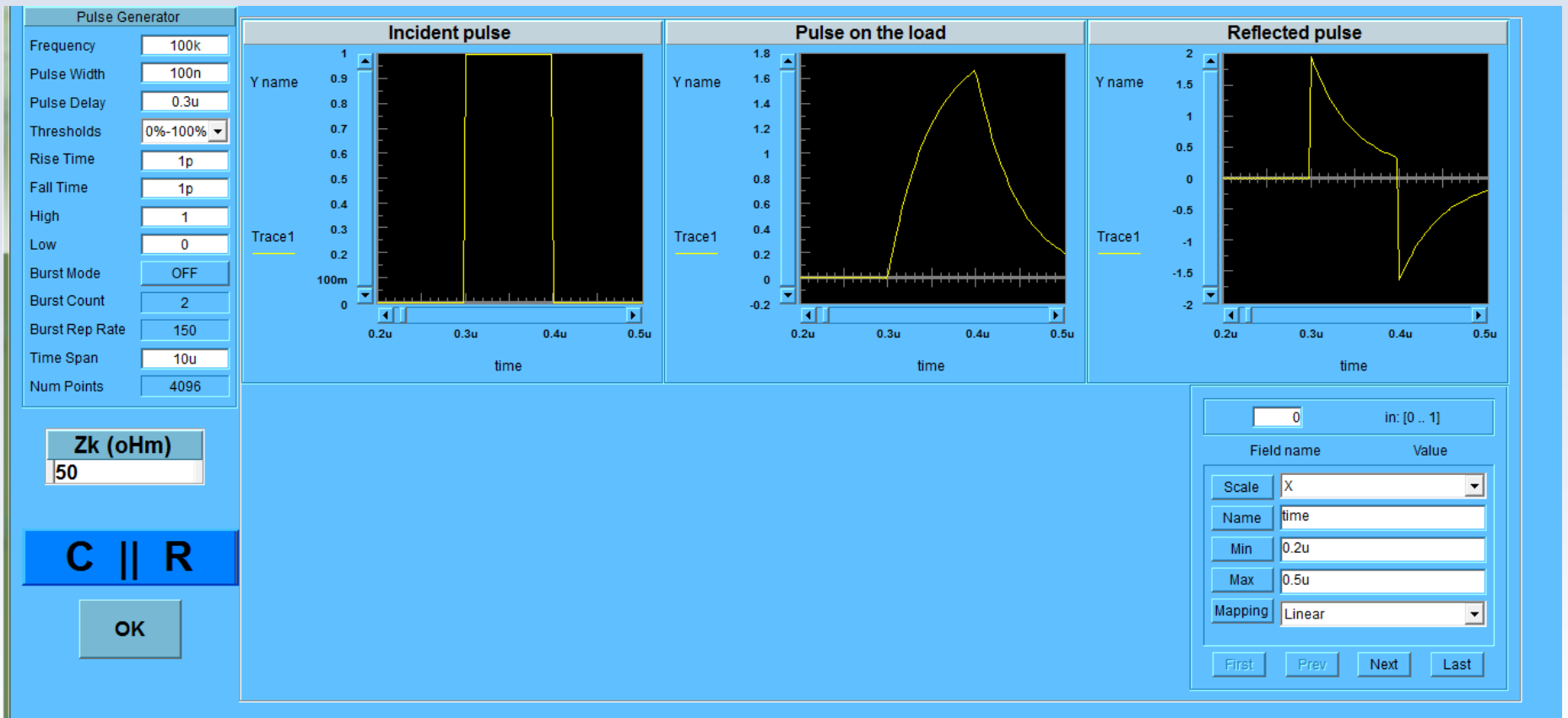


Location:

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- Transmission line. Unknown load simulation**



Location:

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